Data Science in Aerospace

Random Variables

ESCOLA
DE GESTÃO,
ENGENHARIA
E AERONÁUTICA

1 Inspection

An aviation company has 7 inspectors, 2 of whom are female. For a random inspection of an aircraft, two inspectors are selected at random. Let Y denote the number of female inspectors selected.

- Define the sample space for the experiment, designating the inspectors as A, B, C, D, E (males) and F, G (females).
- Define the value of the random variable Y for each element of the sample space.
- Define the probability and distribution functions of the random variable Y. Represent both functions in tabular form and using bar diagrams.
- Calculate the expected value and standard deviation of the random variable Y.

2 Aircraft Maintenance Check

An airport maintenance facility has a fleet of 10 aircraft, 3 of which are fully operational and 7 require maintenance. Three aircraft are selected at random without replacement for a safety inspection.

- What is the probability that at most one fully operational aircraft is selected?
- If exactly one fully operational aircraft was selected in the three inspections, what is the probability that it was selected during the second inspection?
- Let X be the random variable representing the number of fully operational aircraft selected in the three inspections. Determine:

  - The probability mass function (PMF) of X.
  - The cumulative distribution function (CDF) of X.
  - The expected value and variance of X.

- Answer first three points again, assuming that the three inspections were conducted with replacement.

3 Engine Testing

At an airline maintenance facility, there are three identical aircraft engines of the same model that operate independently. The probability of each engine failing during a given time period is 0.1. Let X be the random variable representing the number of engines that are still operational at the end of this time period. Determine:

- The probability mass function of X.
- The cumulative distribution function of X.
- The expected value, mode, median, and variance of X.

4 Airline Booking Service

At an airline ticket counter, the time interval Δt (in minutes) between any two consecutive flight reservation calls is modeled by the following probability density function:

$$f(\Delta t) = \begin{cases} e^{-\Delta t}, & \text{if } \Delta t \geq 0, \\ 0, & \text{if } \Delta t < 0. \end{cases}$$

- Calculate $P(\Delta t > 2)$.
- Calculate $P(\Delta t > 3)$.
- Calculate the conditional probability $P(\Delta t > 3 \mid \Delta t > 1)$.

5 Flight Altitude

Let X represent the altitude (in kilometers) of an aircraft during a specific flight phase. The probability density function of X is given by:

$$f(x) = \begin{cases} k(9x - 6x^2 + x^3), & \text{if } 0 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

- Determine the value of k to ensure $f(x)$ is a valid probability density function. Plot the function $f(x)$.
- Calculate the probabilities $P(X \leq 1.5)$, $P(X \geq 2.0)$ and $P(1.0 \leq X \leq 2.5)$.
- Compute the cumulative distribution function $F(x)$, and plot it graphically.

6 Training Sessions

Consider a discrete random variable X, representing the number of successful landings by a pilot during a training session, with the following probability function:

$$P(X = x) = \begin{cases} ax & \text{if } x \in \{1, 2, 3\}, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a real constant.

- Determine the value of a.
- Determine the cumulative distribution function (CDF) of X.
- Calculate the mode, median, and expected value of X.

7 Fuel Efficiency

An aircraft's fuel efficiency (in miles per gallon) can be considered a random variable X with the probability density function given by:

$$f(x) = \begin{cases} \frac{3}{5}10^{-5}x(100 - x), & \text{if } 0 \leq x \leq 100, \\ 0, & \text{otherwise.} \end{cases}$$

Suppose the profit L obtained from operating the aircraft (per flight hour) depends on the fuel efficiency according to the relationship:

$$L = C_1 + C_2 X,$$

where C_1 and C_2 are constants.

- Calculate the expected value and variance of the profit per flight hour.

8 Maintenance Process

Assume that, in a certain aviation maintenance process, the temperature of the engine oil recorded at the start of each shift in a specific aircraft follows a distribution with an expected value of 153°F and a standard deviation of 7°F.

- Calculate these two parameters of the temperature distribution when expressed in Celsius.