Data Science in Aerospace

Jointly Distributed Random Variables

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1 Landing Performance

An aircraft performs three consecutive flights. Let X be the number of successful landings in the first two flights, and let Y be the number of unsuccessful landings in the last two flights. Assume that both events have equal probability.

- Define the joint probability function of the variables X and Y.
- Define the conditional probability function of Y, given that $X = 1$.
- Calculate the correlation coefficient ρ_{XY}.

2 Departures

An airport monitors the number of flights departing daily from two airlines, Airline X and Airline Y. The joint probability function of the number of flights departing daily is as follows:

X/Y	0	1	2
0	0.12	0.25	0.13
1	0.05	0.30	0.01
2	0.03	0.10	0.01

- Calculate the marginal probability functions of X and Y.
- Calculate the marginal distribution function of X.
- Calculate the probability that, on a given day, Airline Y has more departing flights than Airline X.
- Determine the expected value and variance of the total number of flights departing daily.

3 Flight Parameters

Let X and Y represent random variables related to aviation, where X denotes the wind speed (in knots) and Y denotes the altitude level (in thousands of feet). Their joint probability function is given by:

X/Y	-1	0	1
-1	0	1/4	0
0	1/4	0	1/4
1	0	1/4	0

- Show that $\text{Cov}(X, Y) = 0$ but that X and Y are not independent.

4 Pilot Certification

To qualify for a pilot certification, a candidate must complete two independent tests, A (theoretical test) and B (practical flight test). The performance in each test is classified as insufficient (0), sufficient (1), or excellent (2). The probability of the candidate achieving scores of 0, 1, or 2 in tests A and B is provided in the following table:

Score	Test A	Test B
0	0.2	0.2
1	0.5	0.6
2	0.3	0.2

Consider the random pair (X, Y), where X is the absolute difference between the scores obtained in tests A and B and Y is the sum of the scores obtained in tests A and B.

- Determine the joint probability function of the random pair (X, Y).
- Determine the marginal probability functions of X and Y.
- Determine the cumulative distribution function of X.
- Determine the conditional probability function of X given that $Y = 2$.
- State, with justification, whether X and Y are independent.
- Calculate all conditional probability functions of Y given X.
- Calculate the expected value $E[Y|X = 2]$ and variance $\text{Var}[Y|X = 2]$.

- Calculate $F_{Y|X=0}(y)$.
- Calculate the probability that $Y = 2$ given that $X \cdot Y = 0$.
- Calculate the probability that $X + Y$ is odd.

5 Cruise Flight

Consider two random variables X and Y that represent the altitude (in kilometers) and airspeed (in Mach) of an aircraft, respectively. The joint probability density function is given by:

$$f(x, y) = \begin{cases} 2, & \text{if } (x, y) \in D, \\ 0, & \text{if } (x, y) \notin D, \end{cases}$$

where $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\}$.

- Calculate:
  - The marginal distributions of the altitude (X) and airspeed (Y).
  - The expected values of the altitude (X) and airspeed (Y).
  - The conditional distributions of the altitude (X), given airspeed $Y = y$, and of the airspeed (Y), given altitude $X = x$.
  - The covariance between the altitude (X) and airspeed (Y).

6 Drone Operations

Consider the joint probability density function of two random variables X and Y, where X represents the time (in hours) a drone spends flying, and Y represents the fuel consumption (in liters) during the flight:

$$f(x, y) = \begin{cases} 6(1 - x - y), & \text{if } 0 \leq y \leq 1 - x \text{ and } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

- Are the variables X (flight time) and Y (fuel consumption) independent? Justify your answer.
- Compute the cumulative distribution function (CDF) of the random variable X.
- Determine the conditional density function $f_{X|Y=y}(x)$.

- Calculate $P(X < 1/4 \mid Y = 1/2)$, i.e., the probability that the drone's flight time is less than 1/4 hours given that its fuel consumption is 1/2 liters.
- Calculate $P(X < 3/4 \mid Y > 1/2)$, i.e., the probability that the drone's flight time is less than 3/4 hours given that its fuel consumption exceeds 1/2 liters.

7 Flight Operations

Two airplanes are scheduled to arrive at an airport between 14:00 and 15:00. It is understood that no airplane will wait more than 15 minutes for the other to arrive. Assume that all arrival times within this interval are equally likely.

- What is the probability that the two airplanes will meet at the airport?

8 Wing Design

The wingspan W and the chord length c of an aircraft are random variables with the following parameters:

$$W : \mu_W = 36\,\mathrm{m},\ \sigma_W = 0.5\,\mathrm{m}$$
$$L : \mu_c = 4\,\mathrm{m},\ \sigma_c = 0.1\,\mathrm{m}$$

- Assuming W and c are independent and that the wing is rectangular, estimate the expected value and the standard deviation for the aircraft's wing area.

9 Manufacturing Process

In an aircraft manufacturing process, oscillation systems are produced for a specific type of aircraft. These systems include a damping mechanism with a resistance R and a capacitor with capacitance C. The oscillation period T of the system depends on R and C according to the following relationship:

$$T = \frac{R^3 C^3}{100}.$$

Assume that R and C are random variables with the following parameters:

$$R: \quad \mu_R = 10^6, \quad \sigma_R = 0.03\mu_R,$$
$$C: \quad \mu_C = 10^{-6}, \quad \sigma_C = 0.05\mu_C.$$

Suppose that the resistance and the capacitor incorporated into each oscillation system are selected independently of one another.

- Estimate the expected value and the standard deviation of the oscillation period for these systems.