Data Science in Aerospace

Hypothesis Testing

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1 Engine Verification

An aviation company guarantees that, if their airplane engines are operated under "normal conditions," they have an expected lifespan exceeding 40000 flight hours. A sample of 30 engines operated under the aforementioned conditions yielded the following results:

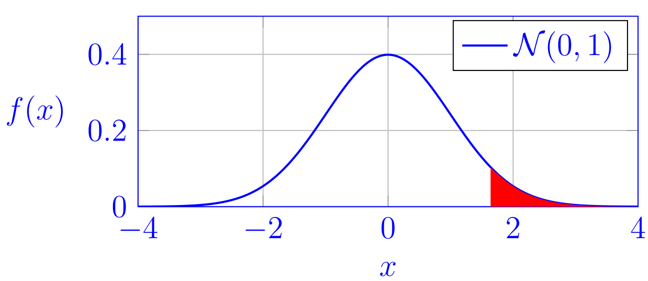
$$\bar{X} = 43200 \text{ flight hours}, \quad S = 8000 \text{ flight hours}.$$

- Test, at a significance level of 5%, whether the engines have the expected lifespan claimed by the manufacturer.

$$H_0 : \mu_X = 40000$$

$$H_a : \mu_X > 40000$$

$$TS = \sqrt{n}\frac{\overline{X} - \mu_0}{S} \sim \mathcal{N}(0,1)$$



$$z_\alpha = z_{0.05} \approx 1.64$$

$$TS = \frac{43200 - 40000}{8000/\sqrt{30}} \approx 2.2 > 1.64$$

H_0 is rejected. We agree with the manufacturer.

$$p(TS > 2.2) \approx 0.01$$

2 Quality Control

Two machines are used in the manufacturing of airplane wing components. The production manager wants to determine if the expected length of the components produced by Machine 1 is greater than that of the components produced by Machine 2. For this purpose, the following data were collected:

- Machine 1 (sample size 10): $\bar{X}_1 = 1.051\,\text{m}$ and $S_1 = 0.021\,\text{m}$
- Machine 2 (sample size 15): $\bar{X}_2 = 1.036\,\text{m}$ and $S_2 = 0.015\,\text{m}$

- Assuming that the lengths of the components follow a Normal distribution, test whether $\mu_1 > \mu_2$ at a significance level of 5%.

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 > \mu_2$$

$$TS = \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{m}}} \sim t_{df}$$

$$df = \frac{\left(\frac{S_1^2}{n} + \frac{S_2^2}{m}\right)^2}{\frac{\left(\frac{S_1^2}{n}\right)^2}{n-1} + \frac{\left(\frac{S_2^2}{m}\right)^2}{m-1}} \approx 15.04$$

$$t_{15}(0.05) \approx 1.75$$

$$TS = \frac{1.051 - 1.036}{\sqrt{\frac{0.021^2}{10} + \frac{0.015^2}{15}}} \approx 1.95 > 1.75$$

H_0 is rejected.

$$p(TS > 1.95) \approx 0.02$$

3 Altimeter Reliability

An aviation equipment manufacturer is considering purchasing altimeters to be installed in a specific type of aircraft, evaluating the possibility of acquiring them from one of two suppliers, A or B. Supplier B offers more expensive altimeters, claiming that they are more reliable. In a test with 9 altimeters from supplier A and 11 altimeters from supplier B, all calibrated to operate at the same altitude, the observed altitudes at which the devices triggered were recorded as follows:

Supplier A: 423, 425, 401, 430, 417, 425, 416, 421, 419

Supplier B: 419, 414, 422, 435, 418, 421, 410, 406, 418, 415, 421

- Verify at a significance level of 5% whether the results confirm the claim that the altimeters provided by supplier B are more reliable.

$$H_0 : \sigma_A^2 = \sigma_B^2$$

$$H_a : \sigma_A^2 > \sigma_B^2$$

$$TS = \frac{S_A^2}{S_B^2} \sim F_{8,10}$$

$$S_A^2 = 68.25$$

$$S_B^2 = 55.69$$

$$F_{8,10}(0.05) \approx 3.07$$

$$TS \approx 1.23 < 3.07$$

H_0 is not rejected.

4 Pilot Performance Test

Seven pilots volunteered to participate in an experiment involving a new training program designed to improve reaction times to auditory signals in the cockpit. Each pilot underwent a test, both before and after completing the training program, where their reaction time (measured in hundredths of a second) to an auditory signal was recorded. It is assumed that, in both situations, the reaction times are Normally distributed. The results of the tests are presented in the table below:

Pilot	A	B	C	D	E	F	G
Reaction Time After Training (s)	17	27	39	27	30	21	36
Reaction Time Before Training (s)	19	22	34	21	27	24	29

- Test, at a significance level of 5%, whether the new training program reduces reaction times to auditory signals as a secondary effect.

5 Fuel System

An aircraft fuel dispensing system is calibrated to fill tanks with 16 liters of fuel. To monitor its performance, 15 fuel tanks were randomly selected during a specific period, and the following fuel volumes (in liters) were recorded:

16.1 15.8 15.9 16.1 15.8 16.2 16.0 15.9
16.0 15.7 15.8 15.7 16.0 16.0 15.8

Assume that the volume of fuel dispensed follows a normal distribution.

- What conclusion can be drawn about the calibration of the fuel dispensing system?
- What evidence supports the evaluation of S^2 under the hypothesis $H_0 : \sigma^2 = 0.25$?

6 Stress Test

A stress test conducted on 12 samples of an aircraft component under compression resulted in the following failure stress values (in kgf/cm^2):

263, 254, 261, 236, 228, 253, 249, 262, 250, 252, 257, 258.

Assume (as is standard in aviation materials testing regulations) that the variable under study follows a normal distribution.

- An engineer wants to determine whether the expected failure stress is not less than 255 kgf/cm^2. What evidence do the data provide regarding this question if a significance level of 5% is used? Justify your answer.
- Knowing that the characteristic failure stress is defined as the value of the variable that has a 95% probability of being exceeded, calculate an estimate of the characteristic failure stress for this aircraft component. Justify the procedure used.

7 Aircraft Engine Performance Analysis

An aviation company collected a sample regarding the fuel efficiency (in miles per gallon, MPG) of two types of aircraft engines. Assume that the fuel efficiency follows a normal distribution and that the two engine types are associated with the same variance. The results obtained were as follows:

Engine Type I	15.01	3.81	2.74	16.82	14.30	13.45	8.75
	9.40	16.84	17.21	2.74	4.91	5.05	9.72
	9.02	12.31	14.10	9.64	10.21	10.34	9.04
	5.02	10.59	11.91	9.44	7.21	11.07	

Engine Type II	10.87	8.07	10.31	11.08	10.84	6.34	10.05
	9.37	8.94	8.78	15.01	6.93	15.91	13.45
	9.37	10.04	10.94	2.04	16.89	14.04	4.32
	10.71	6.84					

- Test whether the expected values for the fuel efficiency of both engine types are equal.
- Construct a 95% confidence interval for the difference between these expected values.

- The manufacturer claims that the variance of fuel efficiency is 4 $(\text{MPG})^2$. Comment on the validity of this claim.

- Let p be the unknown proportion of engines whose fuel efficiency falls below 5 MPG. These engines are considered defective, and the buyer will be compensated if this proportion is greater than 10%. Test the hypothesis that the proportion of defective engines is less than or equal to 10%.

8 Experimental Flight Tests

An aviation company has an experimental airfield where it tests new aircraft designs. A sample of test flights resulted in the following outcomes: 310 flights were successful under clear weather conditions, 109 flights were successful under adverse weather conditions, 100 flights failed under clear weather conditions, and 37 flights failed under adverse weather conditions. In a similar study, an aviation expert predicted the following probabilities based on a simple mathematical model: 56.25% of flights would be successful under clear weather conditions, 18.75% of flights would be successful under adverse weather conditions, 18.75% of flights would fail under clear weather conditions, and 6.25% of flights would fail under adverse weather conditions.

- Are the results from the experimental airfield consistent with the expert's predictions at significance levels of 5% and 1%, respectively?

9 Cruising Altitude

The cruising altitude, in kilometers, of aircraft in a certain fleet is a random variable X. A random sample of 100 aircraft was selected, and their cruising altitudes were recorded. The results are as follows:

Altitude Range (km)	F_i
$(1.595, 1.625)$	5
$(1.625, 1.655)$	18
$(1.655, 1.685)$	42
$(1.685, 1.715)$	27
$(1.715, 1.745)$	8

- Test the fit of a normal distribution with an expected value of 1.675 km and variance 0.029^2 km^2.

$$H_0 : X \sim \mathcal{N}(1.675, 0.029^2)$$

$$H_0 : X \not\sim \mathcal{N}(1.675, 0.029^2)$$

$$TS = \sum_{i=1}^{k} \frac{(X_i - np_i)^2}{np_i} \sim \chi^2_{k-1}$$

$$P_1 = P(1.595 \leq X \leq 1.625) \approx 0.039$$

$$P_2 = P(1.625 \leq X \leq 1.655) \approx 0.203$$

$$P_3 = P(1.655 \leq X \leq 1.685) \approx 0.390$$

$$P_4 = P(1.685 \leq X \leq 1.715) \approx 0.281$$

$$P_5 = P(1.715 \leq X \leq 1.745) \approx 0.076$$

Altitude Range (km)	F_i	p_i	np_i	$(F_i - np_i)^2/(np_i)$
$(1.595, 1.625)$	5	0.039	3.90	0.310
$(1.625, 1.655)$	18	0.203	20.3	0.261
$(1.655, 1.685)$	42	0.390	39.0	0.231
$(1.685, 1.715)$	27	0.281	28.1	0.043
$(1.715, 1.745)$	8	0.076	7.60	0.021
				$TS = 0.866$

$$TS \sim \chi^2_4$$

$$\chi^2_4(0.05) \approx 9.49 > TS$$

H_0 is not rejected.

- Test, at a significance level of 1%, the hypothesis H_0: "X is a random variable with a normal distribution," assuming that the maximum likelihood estimates for μ and σ^2 are the respective moments of the grouped sample.

10 Pilot Training

A major aviation company aims to evaluate the effectiveness of three pilot training programs, labeled as A, B, and C. After completing the training, the performance of 120 pilots was assessed and categorized into three levels: Poor, Satisfactory, and Excellent. The results are summarized in the following table:

Program	**Poor**	**Satisfactory**	**Excellent**
A	6	25	9
B	8	20	7
C	10	30	5

- Test whether the performance of the pilots is independent of the training program they completed. Justify the procedure used.

11 Airspace Control

An aviation authority needs to identify, within its airspace, three types of aircraft operations: commercial flights, private flights, and unmanned aerial vehicles (UAVs). To achieve this, it sends teams to monitor air traffic directly, which incurs high costs. A small company proposes to provide, at a much lower cost, a map of airspace usage obtained through satellite-based analysis. To evaluate whether the company's map is reliable, the aviation authority's experts randomly select 100 instances of air traffic from the region and compare their classification (rows) with the classification provided by the company's map (columns) in the table below:

	Commercial Flights	Private Flights	UAVs
Commercial Flights	16	15	4
Private Flights	14	22	3
UAVs	1	5	20

- Perform a test of independence on this table and comment on your results.

12 Flight Malfunctions

Suppose that the aviation department believes that the probability distribution of the number of malfunctions occurring during a given flight mission in a specific airspace follows a Poisson distribution. The data from 500 such missions are as follows:

Number of malfunctions per mission	Number of missions
0	185
1	180
2	95
3	30
4 or more	10

- Test, at a significance level of 5%, the hypothesis that the random variable in question follows a Poisson distribution with an expected value equal to 1.
- The maximum likelihood estimate of the expected value, numerically evaluated based on the grouped sample, is 0.9845. Is the Poisson model a good choice to describe this dataset?

13 Arrival Time

Suppose we want to test the hypothesis that the time between consecutive aircraft arrivals at an airport follows an exponential distribution with a mean of 100 minutes. That is, the cumulative distribution function for the time between arrivals is given by:

$$F(x) = 1 - e^{-x/100}.$$

If the (ordered) observed times between arrivals from a sample of size 10 are:

66, 72, 81, 94, 112, 116, 124, 140, 145, 155,

what conclusion can be drawn about the hypothesis?

$y_{(j)}$	j/n	$F(y_{(j)})$	$\frac{j}{n} - F(y_{(j)})$	$F(y_{(j)}) - \frac{j-1}{n}$	max
< 0	0	–	–	–	–
66	0.1	0.483	−0.383	0.483	0.483
72	0.2	0.513	−0.313	0.413	0.413
81	0.3	0.555	−0.255	0.355	0.355
94	0.4	0.609	−0.209	0.309	0.309
112	0.5	0.674	−0.174	0.274	0.274
116	0.6	0.687	−0.087	0.187	0.187
124	0.7	0.711	−0.011	0.111	0.111
140	0.8	0.753	0.047	0.053	0.053
145	0.9	0.765	0.135	−0.035	0.135
155	1	0.788	0.212	−0.112	0.212

$$D \approx 0.483$$

$$
\begin{aligned}
D^* &= D(\sqrt{n} + 0.12 + 0.11/\sqrt{n}) && (1)\\
&= 0.483(\sqrt{10} + 0.12 + 0.11/\sqrt{10}) && (2)\\
&\approx 1.60 && (3)
\end{aligned}
$$

$$D^* > d^*_{0.025}(= 1.480)$$

The null hypothesis that the data come from an exponential distribution with mean 100 would be rejected at the 2.5 percent level of significance.