Data Science in Aerospace

Discrete Distributions

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1 Component Verification

An airline company has 10,000 aircraft components ready to be installed on airplanes. Out of these, 500 components are defective and cannot be used. A random inspection is conducted on a sample of 15 components chosen with replacement. The inspection rejects the batch if more than two defective components are found in this sample.

- What is the probability that the batch is rejected?

$$X: \text{ Number of defective components in the sample}$$

$$X \sim B(15, 0.05)$$

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - (P(X = 0) + P(X = 1) + P(X = 2)) \\ &\approx 0.04 \end{aligned}$$

- What is the expected number of defective components in the sample?

$$E[X] = np = 15 \times 0.05 = 0.75$$

$$\mathrm{Var}(X) = np(1 - p) = 15 \times 0.05 \times 0.95 \approx 0.71$$

- Suppose that components are inspected successively (with replacement) until a defective one is found.
  - What is the probability that it will take 4 or more inspections to find a defective component?

$$X: \text{ Number of failures until finding the } r^{\text{th}} \text{ success}$$

$$X \sim BN(1, 0.05)$$

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &\approx 1 - 0.19 \\ &= 0.81 \end{aligned}$$

  - What is the expected number of components inspected?

$$E[X] = \frac{r(1-p)}{p} = \frac{1(1-0.05)}{0.05} = 19$$

2 Aircraft Inspection

In a batch of 500 aircraft components, 50 are defective. From this batch, a random sample is taken with replacement. The batch is rejected if the sample includes more than two defective components.

- What is the probability of rejecting the batch if the sample size is 10.

$$X: \text{ Number of defective components}$$

$$X \sim N(10, 0.1)$$

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &\approx 0.07 \end{aligned}$$

- Calculate the sample size required to ensure that the probability of rejecting the batch is less than 0.05.

$$X \sim N(10, 0.1) \rightarrow P(X \geq 3) \approx 0.07$$
$$X \sim N(9, 0.1) \rightarrow P(X \geq 3) \approx 0.053$$
$$X \sim N(8, 0.1) \rightarrow P(X \geq 3) \approx 0.038$$

- Calculate for samples of size 10, if there are 100 batches under these conditions, what is the expected number of rejected batches?

$$E[X] = np = 10 \times 0.07 = 0.7 \text{ components rejected in 1 batch}$$

$$100 \times 0.7 = 7 \text{ components rejected in 100 batches}$$

3 Component Acquisition

An airline is considering purchasing a batch of 100 aircraft components under the following inspection scheme:

- An inspector randomly selects 5 components and examines them thoroughly.
- The airline will proceed with the purchase if the inspection reveals at most one defective component.

Given that 10 out of the 100 components are defective, determine the probability that the airline proceeds with the purchase, assuming:

- The 5 components are selected randomly one by one with replacement.

$$X: \text{ Number of defective components}$$

$$X \sim N(5, 0.1)$$

$$P(X \leq 1) \approx 0.92$$

- The 5 components are selected randomly one by one without replacement.

$$X: \text{ Number of defective components}$$

$$X \sim H(10, 90, 5)$$

$$P(X \leq 1) \approx 0.92$$

4 Boarding Process

Out of 60,000 passengers in a large airport, 2,000 are scheduled to board a specific flight.

- Write the expression that would allow you to calculate the exact probability that, among 250 passengers randomly selected without replacement from the airport population, fewer than 5 are scheduled to board this flight.

$$X: \text{ Number of people scheduled to board a specific flight}$$

$$X \sim H(2000, 58000, 250)$$

$$\begin{aligned} P(X < 5) = P(X \leq 4) \\ &= \sum_{i=0}^{4} \frac{C(2000, i) \times C(58000, 250 - i)}{C(60000, 250)} \\ &\approx 0.08 \end{aligned}$$

$$\text{If } M > 10N, \text{ } H \text{ can be approximated by } B(N, N/(N+M))$$

5 Quality Inspection

Assume that, at the end of a quality inspection line for aircraft components, 7% of the components are manually inspected by an operator. Additionally, assume that the production rate is one component per minute.

- Calculate the probability that the operator remains idle for ten minutes without performing any inspection.
- Calculate the probability that, in a sequence of six components, the last one is the second component requiring inspection.
- Calculate the average time the operator remains idle without performing any inspection.

6 Maintenance System Failures

An aircraft maintenance system operates continuously and, on average, experiences two system failures per 8-hour shift. For practical purposes, assume that failures are repaired instantly. If a maintenance facility operates 20 aircraft systems simultaneously, calculate:

- The probability of observing 3 failures in the last 10 minutes of a shift.

$$X: \text{ Number of failures in the 20 systems each 10 minutes}$$

$$X \sim \text{Poisson}(0.8)$$

Two failures per 8-hour shift corresponds to 1/24 failures each 10 minutes

$$\lambda = 20 \times \frac{1}{24} = 0.8$$

$$P(X = 3) = \frac{e^{-0.8}(0.8)^3}{3!} \approx 0.04$$

- The expected value and variance of the total number of failures per hour.

Two failures per 8-hour shift corresponds to 1/4 failures per hour

$$E[X] = \lambda = 20 \times \frac{1}{4} = 5$$

$$\text{Var}(X) = \lambda = 5$$

7 Airport Landings

The number of planes landing at an airport within a given time period is a random variable following a Poisson distribution.

- Knowing that the probability of no plane landing during this time period is 1/3, calculate the probability that at least 2 planes land during this time period.

$$L: \text{ Number of landings}$$

$$\begin{aligned} P(L = 0) &= 1/3 \\ \frac{e^{-\lambda}\lambda^0}{0!} &= 1/3 \\ e^{-\lambda} &= 1/3 \\ -\lambda &= \ln(1/3) \\ \lambda &\approx 1.1 \end{aligned}$$

$$L \sim \text{Poisson}(1.1)$$

$$\begin{aligned} P(L \geq 2) &= 1 - P(L \leq 1) \\ &\approx 0.30 \end{aligned}$$

8 Manufacturing Control

In an aircraft manufacturing process, small defects are randomly distributed across the surface of airplane wings. The average density of defects is 0.4 defects per square meter.

- Calculate the probability that, on a wing with dimensions $1.4\,\mathrm{m} \times 3\,\mathrm{m}$, there is at least one defect.

$$D: \text{ Number of defects}$$

0.4 defects per square meter corresponds to 1.68 defects per 4.2 square meters

$$D \sim \text{Poisson}(1.68)$$

$$\begin{aligned} P(D \geq 1) &= 1 - P(D = 0) \\ & 1 - 0.186 \\ & \approx 0.81 \end{aligned}$$

- Calculate the probability that, in a set of 6 wings with dimensions $1.4\,\mathrm{m} \times 3\,\mathrm{m}$ each, at least 4 wings are free of defects.

$$Y: \text{ Number of wings without defects}$$

$$Y \sim B(6, 0.19)$$

$$\begin{aligned} P(Y \geq 4) &= 1 - P(Y \leq 3) \\ &\approx 0.01 \end{aligned}$$

9 Birthday Flight

- Provide an expression that allows you to calculate the exact probability that at least 2 passengers on a flight with 500 passengers have their birthday on Christmas Day (considering a year with 365 days). Obtain an approximate value for this probability using the Poisson distribution.

$$C : \text{Number of people with a birthday on Christmas Day}$$

$$C \sim \text{Poisson}(500/365)$$

$$\begin{aligned} P(C \geq 2) &= 1 - P(C \leq 1) \\ &\approx 0.398 \end{aligned}$$

or

$$C : \text{Number of people with a birthday on Christmas Day}$$

$$C \sim B(500, 1/365)$$

$$\begin{aligned} P(C \geq 2) &= 1 - P(C \leq 1) \\ &\approx 0.398 \end{aligned}$$

10 Pilot Skills

A pilot successfully lands an aircraft with a probability of 4/5.

- In a sequence of 30 landings, calculate approximately the probability that the pilot successfully lands at least 20 times.

$$L : \text{Number of successfully landings}$$

$$L \sim B(30, 0.8)$$

$$P(L \geq 20) \approx 0.974$$