Data Science in Aerospace

Continuous Distributions

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1 Aircraft Altitude Distribution

The cruising altitudes of aircraft in a specific region follow a normal distribution with a mean altitude of 1.70 km and a standard deviation of 0.05 km.

- What is the probability that the cruising altitude of an aircraft exceeds 1.80 km?

$$H : \text{Altitude}$$

$$H \sim \mathcal{N}(1.70, 0.05^2)$$

$$Z = \frac{H - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

$$\begin{aligned} P(H > 1.80) &= P\left(\frac{H - 1.70}{0.05} > \frac{1.80 - 1.70}{0.05}\right) \\ &= P\left(Z > 2\right) \\ &= 1 - P\left(Z \leq 2\right) \\ &\approx 0.98 \end{aligned}$$

- Given that an aircraft is flying at an altitude greater than 1.75 km, what is the probability that it is flying above 1.80 km?

$$P(H > 1.80 \mid H > 1.75) = \frac{P(H > 1.80)}{P(H > 1.75)}$$
$$= \frac{\approx 0.02}{\approx 0.16}$$
$$\approx 0.125$$

- What proportion of aircraft have cruising altitudes between 1.60 km and 1.80 km?

$$P(1.60 < H < 1.80) \approx 0.9545$$

Approximatly 95% of aircraft have cruising altitudes between 1.6 and 1.7 km.

2 Fuselage Control

The inside diameter of the fuselage of an aircraft is a random variable X that follows a normal distribution with a mean of 3 meters and a standard deviation of 0.02 meters. The thickness of the fuselage material Y is another random variable that follows a normal distribution with a mean of 0.3 meters and a standard deviation of 0.005 meters, independent of X.

- Calculate the expected value and the standard deviation of the external diameter of the fuselage.

$$X: \text{ Inside diameter}$$

$$Y: \text{ Fuselage thickness}$$

$$W: \text{ Outer diameter}$$

$$E[W] = E[X + 2Y]$$
$$= E[X] + E[2Y]$$
$$= E[X] + 2E[Y]$$
$$= 3 + 2 \times 0.3 = 3.6 \text{ m}$$

$$\begin{aligned}
\text{Var}(W) &= \text{Var}(X + 2Y) \\
&= \text{Var}(X) + \text{Var}(2Y) + 2\text{cov}(X, Y) \\
&= \text{Var}(X) + 2^2\text{Var}(Y) \\
&= 0.02^2 + 4 \times 0.005^2 = 0.0005 \text{ m}^2
\end{aligned}$$

$$\sigma_W = \sqrt{\text{Var}(W)} \approx 0.02 \text{ m}$$

- Calculate the probability that the external diameter of the fuselage exceeds 3.62 meters.

$$W: \text{ Outer diameter}$$

$$W \sim N(3.6, 0.02^2)$$

$$\begin{aligned}
P(W > 3.62) &= 1 - P(W \leq 3.62) \\
&\approx 0.16
\end{aligned}$$

3 ATC Notifications

The number of flight requests received per day (24 hours) by a small air traffic control center follows a Poisson distribution with a mean of 10.

- Calculate the probability that, on a given day, the center receives no more than 7 flight requests.

$$C: \text{ Number of flight requests on a given day}$$

$$C \sim \text{Poisson}(10)$$

$$P(C \leq 7) \approx 0.22$$

- What is the probability that the interval between two consecutive flight requests exceeds 1 hour?

$$T: \text{ Time interval between two consecutive flight requests (in hours)}$$

$$T \sim \text{Exp}(10/24)$$

$$\begin{aligned} P(T > 1) &= 1 - P(T \leq 1) \\ &= 1 - \int_0^1 \frac{10}{24} e^{-\frac{10}{24}x}\, dx \\ &\approx 0.66 \end{aligned}$$

4 Flight Scheduling

An airport has a total runway capacity of 2800 hours per day. The airport handles 2500 hours daily for commercial airline operations. Additionally, the airport supports 100 private aircraft. Each private aircraft uses an average of 2 hours for flight operations (takeoff, landing, and taxiing), with a standard deviation of 0.5 hours, and an additional 0.5 hours for ground services (refueling and maintenance), with a standard deviation of 0.25 hours.

- Assuming that the operational times for private aircraft are independent and normally distributed, determine the probability that the total runway usage exceeds the airport's capacity, causing delays or disruptions.

5 Critical Diagnostic Test

The variable X represents the time (in minutes) for a specific aviation system to complete a critical diagnostic test. It follows a χ^2 distribution with 19 degrees of freedom.

- Determine the value x_0 such that $P(X < x_0) = 5\%$.

$$X \sim \chi^2_{19}$$

$$P(X < x_0) = 0.05$$
$$P(X \geq x_0) = 0.95 \Rightarrow x \approx 10.12 \text{ minutes}$$

- Determine the probability $P(8.91 < X < 22.72)$.

$$X \sim \chi^2_{19}$$

$$\begin{aligned} P(8.91 < X < 22.72) &= 1 - (P(X \leq 8.91) + P(X \geq 22.72)) \\ &= 1 - (0.025 + 0.250) \approx 0.725 \end{aligned}$$

6 Aircraft Performance and Safety

The variable V represents the deviation in airspeed (in knots) from the expected cruising speed of an aircraft. Assume V follows a t-distribution with 7 degrees of freedom.

- Determine the critical airspeed deviation v_0 such that the probability of exceeding this deviation, $P(V > v_0)$, is 1%.

$$V \sim t_7$$

$$\begin{aligned} P(V > v_0) &= 0.01 \\ v_0 &\approx 3.00 \text{ knots} \end{aligned}$$

- Calculate the probability that the airspeed deviation lies within the range $-1.12 < V < 2.99$ knots.

$$V \sim t_7$$

$$P(-1.12 < V < 2.99) \approx 0.84$$

7 Flight Control System

A flight control system's performance metric U follows an F-distribution with degrees of freedom $F_{24,30}$.

- Determine the threshold value u_0 such that the probability of exceeding this value, $P(U > u_0)$, is 5%.

$$U \sim F_{24,30}$$

$$\begin{aligned} P(U > u_0) &= 0.05 \\ u_0 &\approx 1.89 \end{aligned}$$

- Determine the threshold value u_1 such that the probability of being below this value, $P(U < u_1)$, is 1%.

$$U \sim F_{24,30}$$

$$\begin{aligned} P(U < u_1) &= 0.01 \\ u_1 &\approx 0.39 \end{aligned}$$

8 Aircraft Weight Distribution

An aircraft is designed to carry a maximum of 20 passengers at a time. The total weight capacity of the aircraft is 1300 kg. The passengers belong to a large population where it has been observed that the weight of each passenger follows a normal distribution with a mean of 61 kg and a standard deviation of 10 kg.

- Calculate the probability that the total weight of these 20 passengers exceeds the aircraft's weight capacity.

$$W \sim \mathcal{N}(61, 10^2)$$

$$W_{20} \sim \mathcal{N}(1220, 20 \times 10^2)$$

$$P(W_{20} > 1300) \approx 0.04$$

- Given that there are already 15 passengers onboard with a total weight of 950 kg, and it is expected that 5 more passengers will board to reach full capacity, determine the probability that the total weight of all 20 passengers exceeds the aircraft's weight capacity.

$$W \sim \mathcal{N}(61, 10^2)$$

$$W_{20} \sim \mathcal{N}(1220, 20 \times 10^2)$$

$$W_5 \sim \mathcal{N}(305, 5 \times 10^2)$$

$$\begin{aligned} P(W_5 > 350) &= 1 - P(W_5 \leq 350) \\ &\approx 0.02 \end{aligned}$$

- What is the probability that among the 20 passengers onboard:
  - At most 2 passengers have a weight greater than 85 kg?

$$W \sim \mathcal{N}(61, 10^2)$$

$$P(W > 85) \approx 0.01$$

$$X : \text{ Number of passengers with a weight greater than 85 kg}$$

$$X \sim B(20, 0.01)$$

$$P(X \leq 2) \approx 1.00$$

  - At least 1 passenger has a weight less than 40 kg?

$$W \sim \mathcal{N}(61, 10^2)$$

$$P(W < 40) \approx 0.02$$

$$Y: \text{ Number of passengers with a weight less than 40 kg}$$

$$X \sim B(20, 0.02)$$

$$P(Y \geq 1) \approx 0.67$$

- In your opinion, considering the type of population using the aircraft, is the specified maximum weight capacity appropriate?

Discuss

9 Fuel Efficiency

Let X be a random variable with a normal distribution, mean $\mu = 10$, and variance $\sigma^2 = 4$, representing the fuel efficiency (in liters per kilometer) of an aircraft engine. Suppose the engine is considered efficient if $8 \leq X \leq 12$, and inefficient otherwise.

- What is the probability that an engine is efficient?
- What is the probability that, in a random sample of 10 engines selected with replacement from daily production, at least 2 are inefficient?

10 Landing Scheduling

The time interval, in minutes, between the landing of two airplanes at an airport during peak hours follows a uniform distribution in the interval $[5, 15]$.

- Determine the probability that the waiting time between two airplanes exceeds 8 minutes.
- Given that the last airplane landed 8 minutes ago, what is the probability that the next airplane will take at least another 5 minutes to land? Calculate the expected value of this additional waiting time.
- Assuming that the time intervals between successive airplane landings are independent random variables, calculate an approximate value for the probability that the average of the time intervals over 100 landings exceeds 9 minutes.

11 Airline Profit

A commercial airline operates 10000 flights annually, all under similar operational conditions. The probability of a flight cancellation due to technical issues is 0.006 for each flight. For every scheduled flight, the airline collects a revenue of 12 monetary units (m.u.), and in the event of a cancellation, it incurs a cost of 1000 m.u. (including refunds and penalties). What is the probability that in one year:

- The airline incurs a financial loss?

$$L: \text{ Profit per flight}$$

$$L = \begin{cases} 12 - 1000 & \text{with } p = 0.006 \\ 12 & \text{with } p = 0.994 \end{cases}$$

$$P(L) = \begin{cases} 0.006 & \text{with } l = 12 - 1000 \\ 0.994 & \text{with } l = 12 \end{cases}$$

$$\mu = E[L] = \sum_l lP(L = l) = 6 \text{ m.u.}$$

$$\text{Var}(L) = \sum_l (l - E[X])^2 P(L = l) \approx 5964 \text{ m.u.}^2$$

$$\sigma_L \approx 77.23 \text{ m.u.}$$

$$R = L_1 + L_2 + \ldots + L_n$$

$$R \sim \mathcal{N}(60000, 10000 \times 5964)$$

$$P(R < 0) \approx 0$$

- The airline achieves a profit of at least 40000 m.u.? At least 60000 m.u.? At least 80000 m.u.?

$$R \sim \mathcal{N}(60000, 10000 \times 5964)$$

$$P(R \geq 40000) \approx 0.995$$

$$P(R \geq 60000) = 0.500$$

$$P(R \geq 80000) \approx 0.005$$

12 Cargo Container Analysis

The lengths of the three dimensions of a certain type of cargo container used in aviation follow normal distributions with identical parameters: $\mu = 30\,\text{cm}$ and $\sigma^2 = 25\,\text{cm}^2$.

- Calculate the expected value and standard deviation of the volume of these cargo containers.

$$X, Y, Z \sim \mathcal{N}(30, 25)$$

$$V = XYZ$$

$$\begin{aligned} V \approx V(30,30,30) + \frac{\partial V}{\partial X}(30,30,30)(X-30) + \\ + \frac{\partial V}{\partial X}(30,30,30)(X-30) + \frac{\partial V}{\partial X}(30,30,30)(X-30) \end{aligned}$$

$$\begin{aligned} V &\approx 30^3 + 30^2(X-30) + 30^2(Y-30) + 30^2(Z-30) \\ &= 30^3 + 30^2(X+Y+Z) - 3 \times 30^3 \\ &= 30^2(X+Y+Z) - 2 \times 30^3 \end{aligned}$$

$$\begin{aligned} E[V] &= E[30^2(X+Y+Z) - 2 \times 30^3] \\ &= 30^2(\mu_X + \mu_Y + \mu_Z) - 2 \times 30^3 \\ &= 30^3 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned}\mathrm{Var}(V) &= \mathrm{Var}\left(30^2(X+Y+Z) - 2\times 30^3\right)\\ &= 30^4\mathrm{Var}(X+Y+Z)\\ &= 30^4(\mathrm{Var}(X)+\mathrm{Var}(Y)+\mathrm{Var}(Z))\\ &= 60750000 \text{ cm}^6\end{aligned}$$

$$\sigma_V = \sqrt{\mathrm{Var}(V)} \approx 7794 \text{ cm}^3$$

- Determine the probability that the volume of the containers exceeds 35 liters.

$$V \sim \mathcal{N}(30^3, 7794^2)$$

$$P(V > 35000) \approx 0.15$$

- Repeat the calculations from the previous items, now assuming that all three dimensions of each container are equal (i.e., the container is cubic), with each dimension following a normal distribution with parameters $\mu = 30\,\text{cm}$ and $\sigma^2 = 25\,\text{cm}^2$.

$$X \sim \mathcal{N}(30, 25)$$

$$V = X^3$$

$$V \approx V(30) + \frac{dV}{dX}(30)(X-30)$$

$$\begin{aligned}V &\approx 30^3 + 3\times 30^2(X-30)\\ &= 30^3 + 3\times 30^2 X - 3\times 30^3\\ &= 3\times 30^2 X - 2\times 30^3\end{aligned}$$

$$\begin{aligned}E[V] &= E[3\times 30^2 X - 2\times 30^3]\\ &= 3\times 30^2\mu_X - 2\times 30^3\\ &= 30^3 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(V) &= \mathrm{Var}\left(3 \times 30^2 X - 2 \times 30^3\right) \\ &= \left(3 \times 30^2\right)^2 \mathrm{Var}(X) \\ &= 2250000 \text{ cm}^6\end{aligned}$$

$$\sigma_V = \sqrt{\mathrm{Var}(V)} \approx 1500 \text{ cm}^3$$

$$V \sim \mathcal{N}(30^3, 1500^2)$$

$$P(V > 35000) \approx 0$$