# ISEC Lisboa

## Data Science

**Getting Ready (May 4th, 2026)**

# *FOR TRAINING PURPOSES ONLY*

This sheet of exercises consists of 32 questions (100 %), 9 pages.

(2%) **1.** During successive inspections of fuel line connectors on an aircraft, each connector has a leak with probability $p = 0.3$, independently. Inspections continue until 4 leaking connectors are found. If $N$ is the total number of connectors inspected until the 4$^{\text{th}}$ leak is detected, what is the probability that $N$ equals 10?

- ○ $C(10, 4)\,(0.3)^4\,(0.7)^6$.
- ○ $C(9, 3)\,(0.3)^6\,(0.7)^4$.
- ○ $C(9, 3)\,(0.3)^4\,(0.7)^6$.
- ○ $C(9, 3)\,(0.3)^4\,(0.7)^{10}$.

**2.** Consider the probability density function of a random variable, $V$, representing the wind speed (in $\mathrm{m\,s^{-1}}$) measured at a runway threshold.

$$f(v) = \begin{cases} cv^3 & \text{if } 0 < v < 2, \\ 0 & \text{otherwise.} \end{cases}$$

(2%) (a) The value of the constant $c$ can be found through

- ○ $\int_{-\infty}^{\infty} f(v)\,\mathrm{d}v = 0$.
- ○ $\int_{0}^{v} cv^3\,\mathrm{d}v = 1$.
- ○ $\int_{0}^{\infty} cv^3\,\mathrm{d}v = 0$.
- ○ $\int_{-\infty}^{\infty} f(v)\,\mathrm{d}v = 1$.

(2%) (b) Which expression allows finding the value of $w$ such that the probability of the wind speed being below $w$ equals $0.10$?

- ○ $\int_{w}^{\infty} f(v)\,\mathrm{d}v = 0.10$.
- ○ $\int_{-\infty}^{w} f(v)\,\mathrm{d}v > 0.10$.
- ○ $\int_{-\infty}^{w} f(v)\,\mathrm{d}v = 0.10$.
- ○ $\int_{w}^{\infty} f(v)\,\mathrm{d}v < 0.10$.

(2%) (c) Which expression correctly gives the expected value $E[V]$ of the wind speed?

- ○ $\int_{-\infty}^{v} f(v)\,\mathrm{d}v$.
- ○ $\int_{-\infty}^{\infty} (v - E[V])^2\,f(v)\,\mathrm{d}v$.
- ○ $\int_{-\infty}^{\infty} v^2\,f(v)\,\mathrm{d}v$.
- ○ $\int_{-\infty}^{\infty} v\,f(v)\,\mathrm{d}v$.

(2%) **3.** During flight operations, consider the events: $A =$ *"an aircraft declares a fuel emergency"* and $B =$ *"the aircraft was forced to divert to an alternate airport"*. What is the probability that an aircraft was forced to divert, given that it declared a fuel emergency?

- ◯ $\dfrac{P(A \cap B)}{P(A)}$.
- ◯ $\dfrac{P(A \cap B)}{P(B)}$.
- ◯ $\dfrac{P(B)}{P(A)}$.
- ◯ $P(A \cap B) \cdot P(A)$.

(2%) **4.** At an airport, consider the events: $A$: *"flight departs with a delay"*, $B$: *"flight is diverted to an alternate airport"*. Write, using only set operations (union, intersection and complement), the event $E$: *"the flight was delayed but was not diverted"*.

- ◯ $E = A \cap \overline{B}$.
- ◯ $E = A \cup \overline{B}$.
- ◯ $E = \overline{A} \cap B$.
- ◯ $E = \overline{A \cup B}$.

(2%) **5.** The number of engine parameter exceedances recorded by an onboard health monitoring system can be modelled by a Poisson random variable with mean rate $\lambda = 3$ exceedances per hour. What is the probability of exactly 2 exceedances occurring in a 20-minute interval?

- ◯ $e^{-3}\,\dfrac{3^2}{2!}$.
- ◯ $e^{-1}\,\dfrac{1^3}{3!}$.
- ◯ $1 - e^{-1}\,\dfrac{1^2}{2!}$.
- ◯ $e^{-1}\,\dfrac{1^2}{2!}$.

(2%) **6.** For a sorted dataset with an even number $n$ of observations, the median is the average of the values at positions

- ◯ $n/2$ and $n/2 + 1$.
- ◯ $(n-1)/2$ and $(n+1)/2$.
- ◯ $n/2$ and $n/2 - 1$.
- ◯ $(n+1)/2$ and $(n+3)/2$.

(2%) **7.** A Pearson correlation coefficient of $r = -0.92$ between two variables indicates

- ◯ a weak negative linear correlation.
- ◯ a strong positive linear correlation.
- ◯ a strong causal relationship between the two variables.
- ◯ a strong negative (inverse) linear correlation.

(2%) **8.** In a right-skewed (positively skewed) data distribution, which of the following relations between the mean and the median is typically expected?

- ◯ $\bar{x} < \text{median}$.
- ◯ $\bar{x} = \text{mode}$.
- ◯ $\bar{x} > \text{median}$.
- ◯ $\bar{x} = \text{median}$.

**9.** At a given moment, an airport is operating runway 20, which has 4 available STARs for arriving aircraft.

(2%) (a) An arrival sequence for 8 distinct aircraft is to be created, where each aircraft is freely assigned one of the 4 available STARs. How many possible arrival sequences can be formed in this scenario?

- ◯ $4!$.
- ◯ $4^8$.

- ◯ $P(8,4)$.
- ◯ $8! \times 4$.

(2%) (b) Among these 8 aircraft, 2 are in the *super heavy* category (A380 type). To ensure adequate mitigation of *wake turbulence* separation, these two must always be placed at the end of the arrival sequence. How many distinct arrival sequences are possible?

- ◯ $6 \times 2 \times 4$.
- ◯ $P(8,6) \times P(8,2)$.
- ◯ $8! \times 4!$.
- ◯ $6! \times 2!$.

**10.** A monitoring system detects the presence of a fault in an aircraft's navigation unit with a probability of $0.95$, if the fault exists. If no fault exists, the system correctly identifies its absence with a probability of $0.75$. The probability of a navigation unit having this fault is $0.1$. Calculate the probability of:

(2%) (a) The monitoring system triggering an alert, i.e., detecting the presence of a fault.

- ◯ $P(+) = 1 - P(+|F) - P(+|\overline{F}) = 1 - 0.95 - 0.25$.
- ◯ $P(+) = P(+|F)\,P(F) + P(-|\overline{F})\,P(\overline{F}) = 0.95 \cdot 0.1 + 0.75 \cdot 0.9$.
- ◯ $P(+) = P(+|F) + P(+|\overline{F}) = 0.95 + 0.25$.
- ◯ $P(+) = P(+|F)\,P(F) + P(+|\overline{F})\,P(\overline{F}) = 0.95 \cdot 0.1 + 0.25 \cdot 0.9$.

(2%) (b) The navigation unit actually having a fault, given that the monitoring system triggered an alert.

- ◯ $P(F \cap +) = P(+|F)\,P(F)$.
- ◯ $P(F|+) = \dfrac{P(+|F)\,P(F)}{P(+|F)\,P(F) + P(+|\overline{F})\,P(\overline{F})}$.
- ◯ $P(F|+) = P(F)\,P(+)$.
- ◯ $P(F \cap +) = \dfrac{P(F)}{P(+)}$.

(2%) (c) $E$: *"The monitoring system produces an incorrect result"*.

- ◯ $P(E) = 1 - P(+)$.
- ◯ $P(E) = P(F \cap +) + P(\overline{F} \cap -) = P(F)\,P(+|F) + P(\overline{F})\,P(-|\overline{F})$.
- ◯ $P(E) = P(F \cap -) + P(\overline{F} \cap +) = P(F)\,P(-|F) + P(\overline{F})\,P(+|\overline{F})$.
- ◯ $P(E) = P(+|F) + P(-|\overline{F})$.

**11.** Four people are selected at random and without replacement from a group of 5 air traffic controllers, 3 safety inspectors, and 7 maintenance technicians.

(2%) (a) What is the probability that all four selected are maintenance technicians?

- ◯ $\dfrac{7 \times 6 \times 5 \times 4}{15^4}$.
- ◯ $\left(\dfrac{7}{15}\right)^4$.
- ◯ $\dfrac{C(7,4)}{C(15,3)}$.
- ◯ $\dfrac{C(7,4)}{C(15,4)}$.

(2%) (b) Let $I$ be the event *"the first selected was a safety inspector"* and $A$ the event *"the remaining three selected are air traffic controllers"*. What is the probability of $A|I$?

- ◯ $\dfrac{C(5,3)}{C(14,3)}$.
- ◯ $\dfrac{C(5,3)}{C(15,3)}$.
- ◯ $\dfrac{5 \times 4 \times 3}{14 \times 13 \times 12}$.
- ◯ $\dfrac{5}{14}$.

(2%) (c) What is the probability that the four selected all belong to different roles, with exactly one air traffic controller, one safety inspector, one maintenance technician, and one person from any remaining role?

- ◯ $\dfrac{5 \times 3 \times 7 \times 12}{C(15,4)}$.

○ $\frac{C(5,1)+C(3,1)+C(7,1)}{C(15,4)}$.

○ $\frac{5\times 3\times 7}{C(15,3)}$.

○ $\frac{5\times 3\times 7\times 12}{15\times 14\times 13\times 12}$.

(2%) **12.** An air navigation service provider records daily the number of sector handoffs ($S$) and the number of conflict alerts ($C$) in a control centre. The joint probability function is:

| $S\backslash C$ | 1 | 2 |
|---|---|---|
| 1 | 0.20 | 0.10 |
| 2 | 0.15 | 0.35 |
| 3 | 0.12 | 0.08 |

with $\mathrm{E}[S]=1.88$ and $\mathrm{E}[C]=1.53$. How can we calculate the covariance $\mathrm{Cov}(S,C)$?

○ $\mathrm{Cov}(S,C)=(1-1.88)(1-1.53)\cdot 0.20+(1-1.88)(2-1.53)\cdot 0.10+(2-1.88)(1-1.53)\cdot 0.15$ $+(2-1.88)(2-1.53)\cdot 0.35+(3-1.88)(1-1.53)\cdot 0.12$.

○ $\mathrm{Cov}(S,C)=(1-1.88)(1-1.53)\cdot 0.20+(1-1.88)(2-1.53)\cdot 0.10+(2-1.88)(1-1.53)\cdot 0.15$ $+(2-1.88)(2-1.53)\cdot 0.35+(3-1.88)(1-1.53)\cdot 0.12+(3-1.88)(2-1.53)\cdot 0.08$.

○ $\mathrm{Cov}(S,C)=\mathrm{E}[S\cdot C]+\mathrm{E}[S]\cdot\mathrm{E}[C]$.

○ $\mathrm{Cov}(S,C)=(1-1.88)^2\cdot 0.20+(1-1.88)^2\cdot 0.10+(2-1.88)^2\cdot 0.15$ $+(2-1.88)^2\cdot 0.35+(3-1.88)^2\cdot 0.12+(3-1.88)^2\cdot 0.08$.

**13.** At an airline with 800 crew members, 180 are in favour of a new rest time regulation.

(2%) (a) Considering a random sample of 25 crew members, without replacement, which expression correctly gives the probability that fewer than 6 crew members are in favour of the new regulation?

○ $P(X<6)=\frac{C(180,5)\times C(620,20)}{C(800,25)}$.

○ $P(X<6)=\sum_{k=0}^{5} C(25,k)\left(\frac{180}{800}\right)^k\left(\frac{620}{800}\right)^{25-k}$.

○ $P(X<6)=\sum_{k=0}^{5}\frac{C(180,k)\times C(620,25-k)}{C(800,25)}$.

○ $P(X<6)=1-\sum_{k=0}^{5}\frac{C(180,k)\times C(620,25-k)}{C(800,25)}$.

(2%) (b) A union representative is interviewing crew members one by one to assess opinions on the regulation. What is the expected number of interviews needed until finding three crew members in favour?

○ $\mathrm{E}[T]=\frac{1}{180/800}$.

○ $\mathrm{E}[T]=\frac{3}{180/800}$.

○ $\mathrm{E}[T]=\frac{3\,(1-180/800)}{180/800}$.

○ $\mathrm{E}[T]=3\cdot\frac{180}{800}$.

(2%) **14.** For a LEAP-1A engine, consider the events $V=$ *"the engine suffers an oil pressure drop during the next 50 flight hours"* and $W=$ *"a low oil pressure warning light illuminates"*. It is known that $P(V)=0.03$, $P(W|V)=0.90$ and $P(W|\overline{V})=0.06$. What is the probability that the engine suffered an oil pressure drop, given that the warning light illuminated?

○ $0.90\times 0.03$.

○ $\frac{0.90\times 0.03}{0.90\times 0.03+0.06\times 0.97}$.

○ $\frac{0.03\times 0.06}{0.90\times 0.03+0.06\times 0.97}$.

○ $\frac{0.90\times 0.03}{0.90\times 0.03+0.06\times 0.03}$.

**15.** In an engine certification programme, each test cycle is considered a "pass" with constant probability $p$, independently of the others. A chief engineer wants to know how many test cycles are needed until 3 successful passes are observed.

(2%) (a) If $X$ is the total number of test cycles observed until the third pass occurs, which of the following correctly describes the distribution of $X$ and its probability mass function?

- ◯ Negative binomial: $P(X = x) = \binom{x-1}{2} p^3(1-p)^{x-3}, \quad x = 3, 4, 5, \dots$
- ◯ Binomial: $P(X = x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, \dots, n.$
- ◯ Geometric: $P(X = x) = p(1-p)^{x-1}, \quad x = 1, 2, 3, \dots$
- ◯ Poisson: $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \dots$

(2%) (b) What is the expected total number of test cycles until the third pass occurs, and what is its variance?

- ◯ $\mathrm{E}[X] = 3(1-p)/p, \quad \mathrm{Var}(X) = 3(1-p)/p^2.$
- ◯ $\mathrm{E}[X] = 3/p^2, \quad \mathrm{Var}(X) = 3(1-p)/p.$
- ◯ $\mathrm{E}[X] = 1/p, \quad \mathrm{Var}(X) = (1-p)/p^2.$
- ◯ $\mathrm{E}[X] = 3/p, \quad \mathrm{Var}(X) = 3(1-p)/p^2.$

(2%) **16.** What is the formula for the sample variance $s^2$ for $n$ values $x_k$ with sample mean $\bar{x}$?

- ◯ $s^2 = \dfrac{1}{n-1}\displaystyle\sum_{k=1}^{n} x_k^2.$
- ◯ $s^2 = \dfrac{1}{n}\displaystyle\sum_{k=1}^{n}(x_k - \bar{x})^2.$
- ◯ $s^2 = \sqrt{\dfrac{1}{n-1}\displaystyle\sum_{k=1}^{n}(x_k - \bar{x})^2}.$
- ◯ $s^2 = \dfrac{1}{n-1}\displaystyle\sum_{k=1}^{n}(x_k - \bar{x})^2.$

**17.** At a busy airport, consider the next 40 aircraft pushback operations on a given apron during peak hours. In each operation, a runway incursion alert may be triggered with constant probability $p$. The operations can be assumed to be independent.

(2%) (a) If $X$ is the number of incursion alerts in the 40 operations, which condition is not required for $X$ to follow a binomial distribution?

- ◯ Each operation results in either an alert or no alert.
- ◯ The operations are independent of each other.
- ◯ The number of alerts $X$ must be large.
- ◯ The probability $p$ is constant across all operations.

(2%) (b) What are the expected number of alerts $\mathrm{E}[X]$ and the variance $\mathrm{Var}(X)$?

- ◯ $\mathrm{E}[X] = \dfrac{1}{p}, \quad \mathrm{Var}(X) = \dfrac{1-p}{p^2}.$
- ◯ $\mathrm{E}[X] = 40(1-p), \quad \mathrm{Var}(X) = 40p(1-p).$
- ◯ $\mathrm{E}[X] = 40p, \quad \mathrm{Var}(X) = 40p.$
- ◯ $\mathrm{E}[X] = 40p, \quad \mathrm{Var}(X) = 40p(1-p).$

**18.** A certification flight test can result in two outcomes: pass or fail. The probability of passing is $0.8$. The cost of a single test is 8 k€ if it passes and 20 k€ if it fails. The test is repeated 15 times independently. Let $X$ be the random variable counting the number of tests that pass.

(2%) (a) What is the probability of obtaining fewer than 12 tests passing?

- ◯ $P(X < 12) = 1 - \displaystyle\sum_{k=0}^{11} C(15, k)\,(0.8)^k\,(0.2)^{15-k}.$
- ◯ $P(X < 12) = \displaystyle\sum_{k=12}^{15} C(15, k)\,(0.8)^k\,(0.2)^{15-k}.$

○ $P(X < 12) = \sum_{k=0}^{11} C(15, k)\,(0.8)^k\,(0.2)^{15-k}$.

○ $P(X < 12) = C(15, 11)\,(0.8)^{11}\,(0.2)^4$.

(2%) (b) Let $C$ be the total cost (in k€) of the 15 tests. Calculate the expected value and standard deviation of $C$.

○ $C = 8X + 20(15-X)$, $\mathrm{E}[C] = 156$ k€, $\sqrt{\mathrm{Var}(C)} = \sqrt{345.6}$ k€.

○ $C = 20X + 8(15-X)$, $\mathrm{E}[C] = 264$ k€, $\sqrt{\mathrm{Var}(C)} = \sqrt{345.6}$ k€.

○ $C = 8X + 20X$, $\mathrm{E}[C] = 336$ k€, $\sqrt{\mathrm{Var}(C)} = \sqrt{345.6}$ k€.

○ $C = 8X + 20(15-X)$, $\mathrm{E}[C] = 156$ k€, $\sqrt{\mathrm{Var}(C)} = \sqrt{28.8}$ k€.

(2%) (c) What is the probability that the total cost $C$ is less than 180 k€?

○ $P(C < 180) = P(X > 10) = \sum_{k=10}^{15} C(15, k)\,(0.8)^k\,(0.2)^{15-k}$.

○ $P(C < 180) = P(X < 10) = \sum_{k=0}^{9} C(15, k)\,(0.8)^k\,(0.2)^{15-k}$.

○ $P(C < 180) = P(X > 10) = \sum_{k=11}^{15} C(15, k)\,(0.8)^k\,(0.2)^{15-k}$.

○ $P(C < 180) = P(X \geq 10) = \sum_{k=10}^{15} C(15, k)\,(0.8)^k\,(0.2)^{15-k}$.

(2%) **19.** An aircraft transponder code (squawk) consists of 4 digits, each ranging from 0 to 7. Using the generalised basic counting principle, how many distinct squawk codes are possible?

○ $8 \times 8 \times 8 \times 8 = 8^4 = 4096$.

○ $4 \times 8 = 32$.

○ $8 + 8 + 8 + 8 = 32$.

○ $8^4 - 1 = 4095$ (excluding the code $0000$).

(2%) **20.** In an avionics sensor production line, consider the events $A =$ *"a produced sensor is faulty"* and $B =$ *"the automated inspection system raises an alert for the sensor"*. Which of the following expressions represents the probability that the sensor is truly faulty, given that the inspection system raised an alert?

○ $P(B|A)\,P(A)$.

○ $\dfrac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\overline{A})\,P(\overline{A})}$.

○ $\dfrac{P(B|A)\,P(A)}{P(B)}$.

○ $\dfrac{P(A)\,P(B)}{P(B|A)}$.

(2%) **21.** In an aircraft overhaul, $U$ is the time (in hours) to overhaul the landing gear system and $V$ the time (in seconds) to run a brake pressure test, with $U, V$ independent. The landing gear overhaul is expected to take 6 hours with a variance of 2 hours$^2$, while the brake pressure test is expected to take 120 seconds with a standard deviation of 15 seconds. The total operation time is $T = 3U + V$, in hours. What are $\mathrm{E}[T]$ and $\mathrm{Var}(T)$?

○ $\mathrm{E}[T] = 3 \times 6 + 120,\ \mathrm{Var}(T) = 9 \times 2 + 15^2$.

○ $\mathrm{E}[T] = 3 \times 6 + 120/3600,\ \mathrm{Var}(T) = 9 \times 2 + 15/3600$.

○ $\mathrm{E}[T] = 3 \times 6 + 120/3600,\ \mathrm{Var}(T) = 9 \times 2 + (15/3600)^2 + 3\,\mathrm{Cov}(U, V)$.

○ $\mathrm{E}[T] = 3 \times 6 + 120/3600,\ \mathrm{Var}(T) = 9 \times 2 + (15/3600)^2$.

**22.** At an avionics repair shop, a batch of 30 altimeter units is ready for quality control, of which 8 have a calibration error exceeding the certified tolerance. Five units are selected at random, without replacement, for a detailed inspection.

(2%) (a) Let $X$ be the number of altimeters with a calibration error exceeding the tolerance in the sample. Which of the following correctly states the distribution of $X$ and justifies the choice?

○ Binomial, because each unit is either defective or not, with constant probability.

○ Hypergeometric, because the sample is drawn without replacement from a finite population.

- ○ Negative binomial, because we are counting units until a fixed number of defective ones is found.
- ○ Poisson, because defective units occur randomly in a continuous stream.

(2%) (b) What are the expected number of defective altimeters $\mathrm{E}[X]$ and the variance $\mathrm{Var}(X)$ in the sample?

- ○ $\mathrm{E}[X] = \frac{5 \times 8}{30}, \quad \mathrm{Var}(X) = 5 \cdot \frac{8}{30} \cdot \frac{22}{30} \cdot \frac{30}{29}.$
- ○ $\mathrm{E}[X] = \frac{5 \times 8}{30}, \quad \mathrm{Var}(X) = 5 \cdot \frac{8}{30} \cdot \frac{22}{30} \cdot \frac{25}{29}.$
- ○ $\mathrm{E}[X] = \frac{5 \times 22}{30}, \quad \mathrm{Var}(X) = 5 \cdot \frac{8}{30} \cdot \frac{22}{30} \cdot \frac{25}{29}.$
- ○ $\mathrm{E}[X] = \frac{5 \times 8}{30}, \quad \mathrm{Var}(X) = 5 \cdot \frac{8}{30} \cdot \frac{22}{30}.$

(2%) **23.** Consider the events $A$ = *"an aircraft experiences a bird strike"* and $B$ = *"the flight departs from a coastal airport"*. If $A$ and $B$ are independent, which of the following statements is correct?

- ○ $P(A \cup B) = P(A)\,P(B).$
- ○ $P(A \cap B) = \emptyset.$
- ○ $P(A|B) = P(B).$
- ○ $P(A|B) = P(A).$

(2%) **24.** During cruise flight, the probability of a pilot requesting a flight level change depends on turbulence. Historical data show that 20% of flights encounter moderate-or-severe turbulence. Under these conditions, the probability of a level change request is $P(R|M) = 0.70$. In smooth or light turbulence conditions, $P(R|S) = 0.05$. What is the total probability of a level change request, $P(R)$?

- ○ $P(R|M)\,P(M) \times P(R|S)\,P(S).$
- ○ $P(R|M)\,P(M) + P(R|S)\,P(S).$
- ○ $20\%.$
- ○ $P(R|M) + P(R|S).$

(2%) **25.** A quality control engineer compares the variability of two sensors with different measurement scales. Which quantity is most appropriate for this comparison, and what is its formula?

- ○ Coefficient of variation: $\mathrm{CV} = \frac{\bar{x}}{s} \times 100\%.$
- ○ Coefficient of variation: $\mathrm{CV} = \frac{s}{\bar{x}} \times 100\%.$
- ○ Coefficient of variation: $\mathrm{CV} = \frac{s^2}{\bar{x}} \times 100\%.$
- ○ Standard deviation: $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}.$

**26.** The number of taxiway incursion reports, $N$, filed per day (24 hours) at a regional airport follows a Poisson distribution with a mean of 15.

(2%) (a) What is the probability that, on a given day, the airport files more than 9 reports?

- ○ $P(N > 9) = 1 - \sum_{k=0}^{9} \frac{e^{-15}\,15^k}{k!}.$
- ○ $P(N > 9) = \frac{e^{-15}\,15^9}{9!}.$
- ○ $P(N > 9) = \sum_{k=0}^{9} \frac{e^{-15}\,15^k}{k!}.$
- ○ $P(N > 9) = \sum_{k=10}^{15} \frac{e^{-15}\,15^k}{k!}.$

(2%) (b) Let $T$ be the time (in hours) between two consecutive incursion reports. What is the expected value $\mathrm{E}[T]$ and the variance $\mathrm{Var}(T)$?

- ○ $\mathrm{E}[T] = 15$ h, $\quad \mathrm{Var}(T) = 15$ h$^2$.
- ○ $\mathrm{E}[T] = \frac{24}{15}$ h, $\quad \mathrm{Var}(T) = \frac{24}{15}$ h$^2$.

○ $\mathrm{E}[T] = \dfrac{15}{24}$ h, $\mathrm{Var}(T) = \left(\dfrac{15}{24}\right)^2$ h$^2$.

○ $\mathrm{E}[T] = \dfrac{24}{15}$ h, $\mathrm{Var}(T) = \left(\dfrac{24}{15}\right)^2$ h$^2$.

(2%) **27.** During the flight testing of a *Glide Slope (GS)* system, vertical deviations (in metres) from the nominal glide path were recorded in a sample of 7 consecutive approaches, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7$. Which of the following expressions correctly gives the sample variance, $s^2$, of these deviations?

○ $\dfrac{\sum_{i=1}^{7}(\varepsilon_i - \bar{\varepsilon})^2}{6}$

○ $\dfrac{\sum_{i=1}^{7}\varepsilon_i^2}{6}$

○ $\dfrac{\sum_{i=1}^{7}(\varepsilon_i - \bar{\varepsilon})^2}{7}$

○ $\sqrt{\dfrac{\sum_{i=1}^{7}(\varepsilon_i - \bar{\varepsilon})^2}{6}}$

**28.** At a busy international airport, runway occupancy violations (aircraft or vehicles entering the runway without clearance) are assumed to occur at a constant average rate of $\lambda = 4$ per day.

(2%) (a) Let $X$ be the number of violations in a given 6-hour period. Which of the following correctly identifies the distribution of $X$ and its parameter?

○ Binomial with $n = 4$ and $p = 1/4$.

○ Poisson with $\lambda^* = 24$.

○ Poisson with $\lambda^* = 4$.

○ Poisson with $\lambda^* = 1$.

(2%) (b) What is the probability of no violations occurring in a 6-hour period?

○ $e^{-4}$.

○ $e^{-1/4}$.

○ $1 - e^{-1}$.

○ $e^{-1}$.

(2%) **29.** During a post-maintenance functional check, each of a group of 12 hydraulic valves passes the pressure test with probability $p = 0.92$, independently. If $X$ is the number of valves that fail the check, what is the probability that at most 1 valve fails?

○ $C(12,1)(0.08)^1(0.92)^{11}$.

○ $1 - C(12,1)(0.08)^1(0.92)^{11}$.

○ $C(12,0)(0.08)^0(0.92)^{12} + C(12,1)(0.08)^1(0.92)^{11}$.

○ $C(12,0)(0.92)^0(0.08)^{12} + C(12,1)(0.92)^1(0.08)^{11}$.

(2%) **30.** In an avionics warehouse, there are 25 navigation modules from a production batch: 7 have a firmware fault and 18 are fully operational. Six modules are selected at random, without replacement, for an audit. If $X$ is the number of faulty modules in the sample, what is the probability of finding exactly 3 faulty modules?

○ $C(6,3)\,(0.28)^3\,(0.72)^3$.

○ $\dfrac{C(7,3) \times C(18,3)}{C(25,3)}$.

○ $\dfrac{C(7,3) \times C(18,3)}{C(25,6)}$.

○ $\dfrac{C(7,3) \times C(18,2)}{C(25,6)}$.

**31.** An airline tracks the daily number of maintenance interventions on two aircraft types, Model $A$ and Model $B$. The joint probability function of the number of daily interventions is:

| $A \backslash B$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0.08 | 0.20 | 0.07 |
| 1 | 0.10 | 0.25 | 0.05 |
| 2 | 0.06 | 0.15 | 0.04 |

(2%) (a) What are the marginal probability functions of $A$ and $B$?

- ○ $p_A(0) = 0.08 + 0.10 + 0.06, \quad p_A(1) = 0.20 + 0.25 + 0.15, \quad p_A(2) = 0.07 + 0.05 + 0.04$ and $p_B(0) = 0.08 + 0.20 + 0.07, \quad p_B(1) = 0.10 + 0.25 + 0.05, \quad p_B(2) = 0.06 + 0.15 + 0.04$.
- ○ $p_A(0) = 0.08, \quad p_A(1) = 0.25, \quad p_A(2) = 0.04$ and $p_B(0) = 0.08, \quad p_B(1) = 0.20, \quad p_B(2) = 0.07$.
- ○ $p_A(0) = 0.08 + 0.20 + 0.07, \quad p_A(1) = 0.10 + 0.25 + 0.05, \quad p_A(2) = 0.06 + 0.15 + 0.04$ and $p_B(0) = 0.08 + 0.10 + 0.06, \quad p_B(1) = 0.20 + 0.25 + 0.15, \quad p_B(2) = 0.07 + 0.05 + 0.04$.
- ○ $p_A(a) = \sum_a p(a,b)$ and $p_B(b) = \sum_b p(a,b)$, yielding the same result for $A$ and $B$.

(2%) (b) What is the marginal cumulative distribution function $F_A(a)$ of $A$?

- ○ $F_A(0) = p_A(0) \cdot p_A(1), \quad F_A(1) = p_A(1) \cdot p_A(2), \quad F_A(2) = p_A(0) \cdot p_A(1) \cdot p_A(2)$.
- ○ $F_A(a) = p_A(a)$ for all $a$.
- ○ $F_A(0) = p_A(1) + p_A(2), \quad F_A(1) = p_A(0) + p_A(2), \quad F_A(2) = p_A(0) + p_A(1)$.
- ○ $F_A(0) = p_A(0), \quad F_A(1) = p_A(0) + p_A(1), \quad F_A(2) = p_A(0) + p_A(1) + p_A(2)$.

(2%) (c) What is the probability that on a given day Model $A$ has at least as many interventions as Model $B$?

- ○ $P(A \geq B) = p(0,0) + p(1,1) + p(2,2) = 0.08 + 0.25 + 0.04$.
- ○ $P(A > B) = p(1,0) + p(2,0) + p(2,1) = 0.10 + 0.06 + 0.15$.
- ○ $P(A \geq B) = p(0,0) + p(1,0) + p(1,1) + p(2,0) + p(2,1) + p(2,2) = 0.08 + 0.10 + 0.25 + 0.06 + 0.15 + 0.04$.
- ○ $P(A \geq B) = 1 - P(B > A) = 1 - (p(0,1) + p(0,2) + p(1,2)) = 1 - (0.20 + 0.07 + 0.05)$.

(2%) (d) What is the expected value of the total number of daily interventions $T = A + B$?

- ○ $\mathrm{E}[T] = \sum_{t=0}^{4} t \cdot p_T(t)$, with $p_T(t) = p_A(t) \cdot p_B(t)$.
- ○ $\mathrm{E}[T] = \sum_{t=0}^{4} t \cdot (p_A(t) + p_B(t))$.
- ○ $\mathrm{E}[T] = (0 \cdot p_A(0) + 1 \cdot p_A(1) + 2 \cdot p_A(2)) + (0 \cdot p_B(0) + 1 \cdot p_B(1) + 2 \cdot p_B(2))$.
- ○ $\mathrm{E}[T] = (0 \cdot p_A(0) + 1 \cdot p_A(1) + 2 \cdot p_A(2)) \times (0 \cdot p_B(0) + 1 \cdot p_B(1) + 2 \cdot p_B(2))$.

(2%) **32.** An airline is assembling a team for a long-haul flight with $N$ available pilots and $M$ available cabin crew members. The team must include 3 pilots (with distinct roles: captain, first officer, and relief pilot) and 4 cabin crew members (with no distinct roles among them). How many distinct teams can be formed?

- ○ $P(N,3) \times C(M,4)$.
- ○ $C(N,3) \times C(M,4)$.
- ○ $C(N,3) \times P(M,4)$.
- ○ $P(N,3) \times P(M,4)$.