Integração Numérica

$$\int_1^{1.6} \frac{2x}{x^2-4}\,dx$$

Utilizar a regra composta dos trapézios para estimar o valor do integral com $n = 4$

$$\int_a^b f(x)\,dx \simeq \frac{h}{2}\left[f(a) + 2\sum_{k=1}^{n-1} f(x_k) + f(b)\right]$$

Passo $h = \dfrac{b-a}{n} = \dfrac{1.6-1}{4} = 0.15$

$$x_i = x_0 + ih \qquad f(a) = f(x_0) \qquad f(b) = f(x_n)$$

x_0	x_1	x_2	x_3	x_4
1	1.15	1.30	1.45	1.6

$$\int_1^{1.6} \frac{2x}{x^2-4}\,dx \simeq \frac{0.15}{2}\Big[f(1) + 2\big(f(1.15) + f(1.30) + f(1.45)\big) + f(1.6)\Big] =$$

$$= \frac{0.15}{2}\left[\frac{2\times1}{1^2-4} + 2\left(\frac{2\times1.15}{1.15^2-4} + \frac{2\times1.30}{1.30^2-4} + \frac{2\times1.45}{1.45^2-4}\right) + \frac{2\times1.6}{1.6^2-4}\right]$$

$$= -0.74$$

Utilizar a regra composta de Simpson (1/3) para estimar o valor do integral com $n = 4$

Tem de ser número par!

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4\sum_{k \text{ ímpar}}^{n-1} f(x_k) + 2\sum_{k \text{ par}}^{n-2} f(x_k) + f(b)\right]$$

Passo $h = \frac{b-a}{n} = \frac{1.6-1}{4} = 0.15$

x_0	x_1	x_2	x_3	x_4
1	1.15	1.30	1.45	1.6

$$\int_1^{1.6} \frac{2x}{x^2-4}\,dx \approx \frac{0.15}{3}\Big[\overset{x_0}{f(1)} + 4\big(\overset{x_1}{f(1.15)} + \overset{x_2}{f(1.45)}\big) + 2\big(\underset{x_3}{f(1.30)}\big) + \underset{x_4}{f(1.6)} \Big] = -0.73$$

$$\frac{2\times 1}{1^2-4}$$

$$\frac{2\times 1.15}{1.15^2-4}$$

$$\frac{2\times 1.45}{1.45^2-4}$$

$$\frac{2\times 1.6}{1.6^2-4}$$

$$\frac{2\times 1.30}{1.30^2-4}$$

Equações Diferenciais

Utilizar o metodo de Euler para resolver o problema de valor inicial

$$\begin{cases} y' = -ty \\ y(0) = 1 \end{cases} \qquad h = 0.1$$

$$\boxed{\begin{aligned} t_0 &= 0 \\ y_0 &= 1 \end{aligned}}$$

$$f(t,y) = -ty$$

$$\boxed{y_{k+1} = y_k + h\,f(t_k, y_k)} \qquad \boxed{t_{k+1} = t_k + h}$$

$t_1 = 0.1 \rightarrow y_1 = y_0 + h\,f(t_0, y_0) = 1 + 0.1 \times (-0 \times 1) = 1$

$t_2 = 0.2 \rightarrow y_2 = y_1 + h\,f(t_1, y_1) = 1 + 0.1 \times (-0.1 \times 1) = 0.99$

$t_3 = 0.3 \rightarrow y_3 = y_2 + h\,f(t_2, y_2) = 0.99 + 0.1 \times (-0.2 \times 0.99) = 0.97$

$t_4 = 0.4 \rightarrow y_4 = y_3 + h\,f(t_3, y_3) = 0.97 + 0.1 \times (-0.3 \times 0.97) = 0.94$

$t_5 = 0.5 \rightarrow y_5 = y_4 + h\,f(t_4, y_4) = 0.94 + 0.1 \times (-0.4 \times 0.94) = 0.90$

A aproximação da Solução $y(t)$ entre 0 e 0.5 é:

t	0	0.1	0.2	0.3	0.4	0.5
y	1	1	0.99	0.97	0.94	0.90

Utilizar o metodo de RK de 4ª ordem para resolver o problema de valor inicial

$$\begin{cases} y' = e^t y^2 + e^3 \\ y(2) = 4 \end{cases} \qquad \begin{matrix} t_0 = 2 \\ y_0 = 4 \end{matrix} \qquad \begin{matrix} t_5 = 5 \\ y_5 = ? \end{matrix}$$

$$h = \frac{t_5 - t_0}{5} = \frac{5-2}{5} = 3/5$$

$$\boxed{y_{k+1} = y_k + \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)} \qquad \boxed{t_{k+1} = t_k + h}$$

$t_0 = 2 \rightarrow y_0 = 4$

calculados no ponto anterior

$$t_1 = 13/5 \rightarrow y_1 = \overbrace{y_0}^{} + \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$$

$$= 4 + \frac{1}{6}\left(82.99 + 2\times 12398 + 2\times 2.3\times 10^8 + 4.2\times 10^{17}\right) \approx 7\times 10^{16}$$

C.A.

$$K_1 = h f(t_0, y_0) = 3/5 \times \left(e^2 \times 4^2 + e^3\right) \simeq 82.99$$

$$K_2 = h f\left(t_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) \simeq 3/5 \times \left(e^{2+3/10} \times (4 + 82.99/2)^2 + e^3\right) \simeq 12398$$

$$K_3 = h f\left(t_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) \simeq 3/5 \times \left(e^{2+3/10} \times \left(4 + \frac{12398}{2}\right)^2 + e^3\right) \simeq 230295447 = 2.3\times 10^8$$

$$K_4 = h f(t_0 + h, y_0 + K_3) \simeq 3/5 \times \left(e^{2+3/5} \times (4 + K_3)^2 + e^3\right) \simeq 4.2\times 10^{17}$$

$230295447 = 2.3\times 10^8$

$$t_2 = 16/5 \rightarrow y_2 = y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

onde,

$$K_1 = h\, f(t_1, y_1)$$

$$K_2 = h\, f(t_1 + h/2, y_1 + K_1/2)$$

$$K_3 = h\, f(t_1 + h/2, y_1 + K_2/2)$$

$$K_4 = h\, f(t_1 + h, y_1 + K_3)$$

$$\vdots$$

$$t_5 = 5 \rightarrow y_5 = y_4 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$