

Interpolação
Lagrange

	$i=0$	$i=1$	$i=2$	$i=3$
x	x_0	x_1	x_2	x_3
	0	2	3	4
$f(x)=y$	7	11	28	64
	y_0	y_1	y_2	y_3



4 pontos permite a criação de um polinómio de grau 3.

$$P_m(x) = \sum_{i=0}^m l_i(x) f(x_i)$$

$$l_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^m \left(\frac{x - x_j}{x_i - x_j} \right) \quad (0 \leq i \leq m)$$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1} \right) \left(\frac{x - x_2}{x_0 - x_2} \right) \left(\frac{x - x_3}{x_0 - x_3} \right)$$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0} \right) \left(\frac{x - x_2}{x_1 - x_2} \right) \left(\frac{x - x_3}{x_1 - x_3} \right)$$

$$l_2(x) = \left(\frac{x - x_0}{x_2 - x_0} \right) \left(\frac{x - x_1}{x_2 - x_1} \right) \left(\frac{x - x_3}{x_2 - x_3} \right)$$

$$l_3(x) = \left(\frac{x - x_0}{x_3 - x_0} \right) \left(\frac{x - x_1}{x_3 - x_1} \right) \left(\frac{x - x_2}{x_3 - x_2} \right)$$

$$P_3(x) = \sum_{i=0}^3 l_i(x) f(x_i) = \left(\frac{x - x_1}{x_0 - x_1} \right) \left(\frac{x - x_2}{x_0 - x_2} \right) \left(\frac{x - x_3}{x_0 - x_3} \right) f(x_0) +$$

$$+ \left(\frac{x - x_0}{x_1 - x_0} \right) \left(\frac{x - x_2}{x_1 - x_2} \right) \left(\frac{x - x_3}{x_1 - x_3} \right) f(x_1) + \dots$$

$$+ \left(\frac{x-x_0}{x_2-x_0} \right) \left(\frac{x-x_1}{x_2-x_1} \right) \left(\frac{x-x_3}{x_2-x_3} \right) f(x_2) + \left(\frac{x-x_0}{x_3-x_0} \right) \left(\frac{x-x_1}{x_3-x_1} \right) \left(\frac{x-x_2}{x_3-x_2} \right) f(x_3)$$

Substituindo =

$$\left(\frac{x-2}{0-2} \right) \left(\frac{x-3}{0-3} \right) \left(\frac{x-4}{0-4} \right) \times 7 + \left(\frac{x-0}{2-0} \right) \left(\frac{x-3}{2-3} \right) \left(\frac{x-4}{2-4} \right) \times 11 +$$

$$+ \left(\frac{x-0}{3-0} \right) \left(\frac{x-2}{3-2} \right) \left(\frac{x-4}{3-4} \right) \times 28 + \left(\frac{x-0}{4-0} \right) \left(\frac{x-2}{4-2} \right) \left(\frac{x-3}{4-3} \right) \times 64 =$$

$$= \frac{(x-2)(x-3)(x-4)}{-24} \times 7 \dots \text{Se quiser simplificar}$$

Newton (Interpolação)

x	1	3	-2	4	5
y	2	6	-1	-4	2

! Ordenar Dados !

	x_0	x_1	x_2	x_3	x_4
x	-2	1	3	4	5
y	-1	2	6	-4	2

$$P_n(x) = \sum_{i=0}^n a_i \left(\prod_{j=0}^{i-1} (x-x_j) \right)$$

? ? ↓ = ?

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots$$

$$P_4(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2) + a_4(x - x_0)(x - x_1)(x - x_2)(x - x_3)$$

x	y				
x_0	-2	-1	1	$\frac{1}{5}$	-4
x_1	1	2	2	$\frac{1}{5}$	-4
x_2	3	6	-10	-4	3
x_3	4	-4	6	+8	
x_4	5	2			

$\frac{2 - (-1)}{1 - (-2)} = 1 \rightarrow a_1$
 $\frac{-4 - \frac{1}{5}}{4 - (-2)} = -0.7 \rightarrow a_3$
 $0.53 \rightarrow a_4$

$$P_4(x) = -1 + 1x(x+2) + \frac{1}{5}(x+2)(x-1) + (-0.7)(x+2)(x-1)(x-3) + 0.53(x+2)(x-1)(x-3)(x-4)$$

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